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There is a sharp increase in the relative number of degrees of freedom, n, at the deconfining phase transition. If n is proportional to the pressure, then in a Polyakov Loop model of QCD, the energy loss of a fast parton grows relatively quickly, $\propto \sqrt{n}$; in contrast, dilepton production grows slowly, $\propto n^2$.

Experiments indicate that for the central collisions of large nuclei, $A \sim 200$, there are marked changes between energies of $\sqrt{s}/A = 17$ GeV, at the SPS, and 130 GeV, at RHIC [1]. Comparing central AA collisions to pp, the spectrum of semi-hard particles is rather different. At the SPS, in AA the hard p_t spectrum, scaled by the number of binary collisions, is enhanced over pp. At RHIC, the opposite is true: the semi-hard p_t spectrum per nucleon-nucleon collision, is suppressed in central AA, relative either to peripheral AA, or $p\bar{p}$ [2]. This could be the result of "energy loss" [3–5], where a fast colored field loses energy as it passes through a thermal bath. In peripheral AA collisions, secondary hadrons are distributed anisotropically in the transverse momentum p_t [6]. Experimentally, this azimuthal anisotropy increases with p_t until $p_t \sim 2$ GeV, at which point it flattens [7]. This flattening may also be due to energy loss [8].

In the limit of infinitely large nuclei, $A \to \infty$, it is plausible that the initial energy density produced in a central AA collision — at a fixed value of $\sqrt{s}/A$ — evolves into a system in equilibrium at a temperature T. With great optimism, assuming that $A \sim 200$ is near $A = \infty$, one might imagine that the difference between SPS and RHIC is because temperatures reached at RHIC exceed T_c, the critical temperature for QCD.

Thus it is of interest to know how quantities change as one goes through the phase transition. In this paper we give an analysis in terms of the Polyakov Loop model [9–11].

In QCD, there is a large increase in the number of degrees of freedom at the deconfining phase transition. We count degrees of freedom as appropriate for the pressure of free, massless fields at nonzero temperature, so if each boson counts as one, then each fermion counts as 7/8. In the hadronic phase, pions contribute $c_\pi = 3$ ideal degrees of freedom. By asymptotic freedom, at infinite temperature QCD is an Ideal Quark-Gluon Plasma, with $c_{\rm QGP} = 47\,\frac{1}{2}$ degrees of freedom. This is an increase of more than a factor of ten.

To measure the change in the number of degrees of freedom, we introduce the relative pressure, $n(T)$: at a temperature T, this is the ratio of the true pressure, $p(T)$, to that of an Ideal Quark-Gluon Plasma, $p_{\rm IQGP} = c_{\rm QGP}(\pi^2/90)T^4$:

$$n(T) \;\equiv\; \frac{p(T)}{p_{\rm IQGP}} \quad . \tag{1}$$

By asymptotic freedom, QCD is an ideal gas at infinite temperature, and so

$$n(\infty) = 1 \; . \tag{2}$$

For $T < \infty$, corrections to ideality are determined by the QCD coupling constant, $\alpha_s \propto 1/\log(T)$, with $n(T) - 1 \propto -\alpha_s$ [12].

For an *exact* chiral symmetry which is spontaneously broken by the vacuum, about zero temperature the free energy is that of free, massless pions. Thus at zero temperature, the relative pressure is the ratio of the ideal gas coefficients [13]:

$$n(0) \;=\; \frac{c_\pi}{c_{\rm QGP}} \; . \tag{3}$$

At low temperature, corrections to ideality are given by chiral perturbation theory for massless pions, $n(T) - n(0) \sim +(T/f_\pi)^4 \; n(0)$, with f_π the pion decay constant. In QCD, pions are massive, and the relative pressure is Boltzmann suppressed at low temperature, $n(T) \sim \exp(-m_\pi/T)(m_\pi/T)^{5/2}$, so $n(0) = 0$.

Given the great disparity between c_π and $c_{\rm QGP}$, consider an approximation where the hadronic degrees of freedom are neglected relative to those of the deconfined phase [14]. Then the relative pressure vanishes throughout the hadronic phase, $n(T) = 0$ for $T < T_c$. The question is then: how does the relative pressure go from zero at T_c, when deconfinement occurs, to near one at higher T?

This can be answered by numerical simulations of Lattice QCD [15]. Consider first quenched QCD, with pure glue and no dynamical quarks, which is close to the continuum limit [16]. For three colors, the Lattice finds no measurable pressure in the hadronic phase (glueballs are heavy), so our approximation of $n(T) = 0$ when $T < T_c$ is good. $n(T)$ increases quickly above T_c, and is $\sim .8$ by $T \sim 2T_c$. To characterize the change in the relative pressure, consider the ratio of $e - 3p$, where $e(T)$ is the true energy density of QCD, to the energy of an Ideal Quark-Gluon Plasma, $e_{\rm IQGP} = 3\, p_{\rm IQGP}$:

$$\frac{e - 3p}{e_{\rm IQGP}} = \frac{T}{3}\frac{\partial n}{\partial T} \; . \tag{4}$$

Lattice simulations find that this ratio has a sharp "bump" at $\sim 1.1 T_c$, suggesting that the relative pressure changes quickly, when the reduced temperature,

$$t \equiv \frac{T}{T_c} - 1 \; , \tag{5}$$

is small, $t \sim .1$.

The Lattice is more uncertain with dynamical quarks. The pions are too heavy, and it is not near the continuum limit. So far, the Lattice finds that $n(T/T_c)$ is about the same with dynamical quarks as without [15,17]. This suggests that the pure glue theory may be a reasonable guide to how the relative pressure increases above T_c. The approximate universality of $n(T/T_c)$ is remarkable. At present, the Lattice finds no true phase transition in QCD, with T_c smaller by $\approx .6$ than in the quenched theory [15]. Indeed, even the ideal gas coefficients are very different: $c_{\rm QGP}$ is only 16 in the quenched theory, versus $47\,\frac{1}{2}$ in QCD.

The greatest change with dynamical quarks is a small, but measurable, pressure in the hadronic phase. While in the quenched theory $n(T) \approx 0$ for $T < T_c$, with dynamical quarks, although $n(0) \approx 0$, there is a nonzero relative pressure at the critical temperature, with $n(T_c) \approx .1$ [15]. Indeed, with no true phase transition, an approximate T_c can only be defined as the point where the relative pressure increases sharply, reaching $n \approx .8$ by $2T_c$ [15].

The Polyakov Loop model [9–11] is a mean field theory for the relative pressure. In a pure glue theory, the expectation value of the Polyakov Loop, $\ell_0(T)$, behaves like the relative pressure: it vanishes when $T < T_c$, and is nonzero above T_c. Indeed, again by asymptotic freedom, $\ell_0 \to 1$ as $T \to \infty$. The simplest guess for a potential for the Polyakov Loop is:

$$V(\ell) = -\frac{b_2}{2}|\ell|^2 + \frac{1}{4}\left(|\ell|^2\right)^2 \; . \tag{6}$$

Defining ℓ_0 as the minimum of $V(\ell)$ for a given $b_2(T)$, the relative pressure is given by [9–11]:

$$n(T) = -4 V(\ell_0) = \ell_0^4 \; ; \tag{7}$$

$b_2 > 0$ above T_c ($b_2(T) \to 1$ for $t \to \infty$), and < 0 below T_c [18]. Thus if the relative pressure changes when the reduced temperature $t \sim .1$, the change for $\ell_0(T) \sim n^{1/4}$ is even more rapid, within 2.5% of T_c.

For two colors, (6) is a mean field theory for a second order deconfining transition [19]. The ℓ field is real, and so the potential defines a mass, $m_\ell^2 = \partial^2 V/\partial \ell^2$, with

$$m_\ell(T) \propto \ell_0 \sim n^{1/4} \; . \tag{8}$$

This is measured from the two point function of Polyakov loops in coordinate space, $\propto (1/r)\exp(-m_\ell T r)$ as $r \to \infty$.

For three colors, ℓ is a complex valued field, and a term cubic in ℓ appears in $V(\ell)$, $-b_3(\ell^3 + \ell^{*3})/6$. This produces a first order deconfining transition, where ℓ_0 jumps from 0 at T_c^- to $\ell_c = 2b_3/3$ at T_c^+ [10]. The ℓ field has two masses, from its real (m_ℓ) and imaginary ($\widetilde{m}_\ell$) parts. At T_c^+, $m_\ell = \ell_c$; from the Lattice, $m_\ell \sim .1$, [15], which gives $b_3 \sim .15$. (Also, $\widetilde{m}_\ell(T) \propto \sqrt{b_3 \ell} \sim n^{1/8}$.) This small value of b_3 reflects the weakly first order deconfining transition for three colors [15,16].

With dynamical quarks, the hadronic pressure found by the Lattice below T_c can be incorporated by adding a term linear in ℓ to $V(\ell)$, $-b_1(\ell + \ell^*)/2$ [20]. Taking $n(T_c) \sim .1$ [15], and including the pion pressure below T_c, b_1 is negligible, $\sim .01$.

Thinking of ℓ_0 provides a useful way of viewing the deconfining phase transition. For a strongly first order transition — as appears to occur for four or more colors [21] — ℓ_0, jumps from zero below T_c, to a value near one just above T_c. As ℓ_0 is near one, the deconfined phase is presumably well described as a nearly Ideal Quark-Gluon Plasma [22]. In this case, there is a hadronic phase below T_c, and a Quark-Gluon Plasma from T_c immediately on up.

In contrast, for three colors the deconfining transition is weakly first order. As the energy density is discontinuous at T_c, for small t the relative pressure is linear in the reduced temperature,

$$n(T) \;\sim\; 3\, r\, t \; ; \tag{9}$$

here $r \equiv e(T_c^+)/e_{\rm IQGP}(T_c)$ is the ratio of the energies at T_c, in the deconfined phase versus an Ideal Quark-Gluon Plasma. For quenched QCD, $r \sim 1/3$ [16], which gives $n(T) \propto t$, and so $\ell_0(T) \propto t^{1/4}$. Except very near T_c, this simple estimate agrees with more complicated analysis using $b_3 \neq 0$ [10,11]. For example, at only 5% above T_c, this estimate gives $\ell_0 \sim .05^{1/4} \sim .5$. For three colors, then, there is a (non)-Ideal Quark-Gluon Plasma only at temperatures above $\approx 2T_c$; between T_c and $\approx 2T_c$, the Polyakov Loop dominates the free energy, going from $\approx .5$ at $1.05 T_c$ to ≈ 1 by $2T_c$.

The difference between these two scenarios: a strongly first order transition, where $\ell_0(T)$ doesn't change much above T_c, and nearly second order behavior, where $\ell_0(T)$ does change, is in principle observable.

To illustrate how quantities change as the relative pressure turns on, consider energy loss for a fast parton, with a high energy E. Just above T_c, the relative number of degrees of freedom is small, $n(t) \propto t$, and we can perform a low density expansion. Scattering once in a dilute medium gives the radiative energy loss [4,23]:

$$-\frac{dE}{dz} \;=\; \frac{6\,\alpha_s}{\pi}\,\frac{E}{\lambda}\,\log\frac{4E^2}{m_\ell^2} \; ; \tag{10}$$

z is the distance in the medium, and λ is the mean free path [24]. Energy loss is logarithmically divergent in the infrared, cutoff by a mass for static, electric fields. This is provided by a mass for the Polyakov Loop, m_ℓ, as in (8) [25].

The inverse mean free path, λ^{-1}, is approximately the product of the density, ρ, times the elastic cross section, $\sigma_{\rm el}$. We assume that the density is linear in the relative pressure, $\rho(T) \propto n(T)$. The elastic cross section is quadratically infrared divergent; assuming this is cutoff by m_ℓ, $\sigma_{\rm el} \propto \alpha_s^2/m_\ell^2$. In all, since the density vanishes quicker than the elastic cross section diverges, the mean free path also diverges, $\lambda \propto 1/\ell_0^2 \sim n^{-1/2}$. Consequently, radiative energy loss vanishes as $T \to T_c^+$,

$$-\frac{dE}{dz} \propto \ell_0^2 \sim n^{1/2} . \tag{11}$$

The above represents energy loss from scattering once in a dilute medium. As the density increases, multiple scattering becomes important. On average there are $\mathcal{N} = L/\lambda$ scatterings over a length L. We introduce the Landau-Pomeranchuk-Migdal (LPM) energy [4,5], $E_{\rm LPM} \equiv \lambda m_\ell^2$; in our model, this is independent of ℓ_0. In the Bethe-Heitler limit the medium-induced soft radiation is just a sum over contributions from isolated single scatterings [4,5], and scales again like $1/\lambda \sim \ell_0^2 \sim n^{1/2}$. On the other hand, in the LPM regime successive scatterings coherently interfere [4,5]. Integrating the differential radiation intensity distribution over frequency, for jet energies $E/E_{\rm LPM} < \mathcal{N}^2$ the total medium-induced energy loss of the hard jet is again proportional to the inverse mean free path, and so scales as in (11).

On the other hand, for jet energies $E/E_{\rm LPM} > \mathcal{N}^2$, energy loss is (up to logarithms) linear in the relative pressure,

$$-\frac{dE}{dz} = \frac{3\alpha_s}{\pi} \frac{m_\ell^2}{\lambda} L \propto \ell_0^4 \sim n . \tag{12}$$

Compared to (11), there is an additional factor of $m_\ell^2 \sim \ell_0^2 \sim n^{1/2}$. This reflects the transverse momentum broadening of the soft radiation from the hard jet due to multiple collisions. In both of the above cases, independent multiple scattering is assumed, with the range of the potential smaller than the mean free path, $m_\ell^{-1} \ll \lambda$ [4,5]. This is satisfied both perturbatively at extremely high temperature ($\ell_0 \sim 1$, $\alpha_s < 1$) and near T_c^+ ($\ell_0 < 1$).

Our estimates are very rough. A low density expansion works for small ℓ_0. For perturbation theory to be valid, though, $\sigma_{\rm el} \propto \alpha_s^2/m_\ell^2$ should not be too large, so ℓ_0 cannot be too small. Even with the limitations of our approximations, it is clear that since the density vanishes as $T \to T_c^+$, *any* contribution from the deconfined phase vanishes like *some* power of $n(T)$ [26]. For example, the production of thermal (continuum) dileptons is proportional to the density squared [27,28],

$$N_{e^+e^-}(T) \propto n^2 . \tag{13}$$

Above T_c, as the relative pressure increases linearly in the reduced temperature, $n(t) \sim t$, energy loss (11) grows more quickly, $\propto n^{1/2} \sim t^{1/2}$, while dilepton production (13) grows more slowly, $\propto n^2 \sim t^2$.

We can make qualitative predictions. If $\sqrt{s}/A = 130$ GeV corresponds to a temperature above but near T_c, then due to the change $\propto \ell_0^2(T)$, energy loss should be significantly larger at a higher $\sqrt{s}/A$, such as 200 GeV. Conversely, it decreases rapidly at lower $\sqrt{s}/A$.

For a strongly first order transition, implicit in [3–5,8], $\ell_0(T)$ changes little above T_c. Energy loss varies with $\sqrt{s}/A$, but weakly, with the presumably small change in temperature.

Thus we await results from RHIC at both higher, and lower, energies.

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